# MATHEMATICS II

# 029

**29 Oct. 2014 08.30AM - 11.30AM**

REB
Rwanda Education Board

## ADVANCED LEVEL NATIONAL EXAMINATIONS, 2014



**SUBJECT: MATHEMATICS II**

**COMBINATIONS :**

- **MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)**
- **MATHS-COMPUTER SCIENCE-ECONOMICS (MCE)**
- **MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)**
- **MATHS-PHYSICS-COMPUTER SCIENCE (MPC)**
- **MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)**
- **PHYSICS-CHEMISTRY-MATHEMATICS (PCM)**
- **PHYSICS-ECONOMICS-MATHEMATICS (PEM)**

**DURATION: 3 HOURS**

**INSTRUCTIONS:**

1. Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets of paper if provided.
2. Do not open this question paper until you are told to do so.
3. This paper consists of **two** sections: **A** and **B**.
   - **Section A**: Attempt **all** questions. **(55 marks)**
   - **Section B**: Attempt **only three** questions. **(45 marks)**
4. ***Geometrical instruments and silent non-programmable calculators may be used.***

**SECTION A : ATTEMPT ALL QUESTIONS. (55 marks)**

1. The line $4x+3y=-k$ is tangent to the circle $x^2+y^2-4=0$.

   Find the value of $k$. ***(3 marks)***

2. a) Determine the domain of definition of the function f(x) = $\sqrt{4-x^2}+\frac{1}{x}$ ***(1 mark)***

   b) Find the period of the function f(x) = tan $\left(4\pi^2 x-\frac{\pi}{3}\right)$ ***(2 marks)***

3. Solve in $\Re$ the following equation: $\left(x+\sqrt{x}\right)^4-\left(x+\sqrt{x}\right)^2$ = 159,600. ***(3 marks)***

4. a) Write (if possible) the vector $\vec{a}\,(1,6)$ as linear combination of vectors $\vec{u}\,(1,3)$ and $\vec{v}\,(-1,-2)$. ***(2 marks)***

   b) Given $\begin{cases}\vec{v}_1=2\vec{e}_1+\vec{e}_2\\ \vec{v}_2=3\vec{e}_1-4\vec{e}_2\end{cases}$

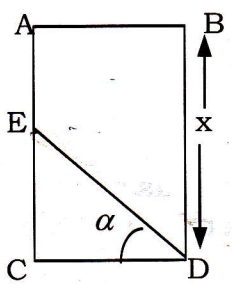
   Write $\vec{e}_1$ and $\vec{e}_2$ as linear combination of vectors $\vec{v}_1$ and $\vec{v}_2$ ***(2 marks)***

5. If A = $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, find $\alpha$ and $\beta$ so that $(\alpha I+\beta A)^2$ = A, where I is the identity matrix. ***(3 marks)***

6. a) Find the Cartesian equation of the plane $\alpha$ which passes through the point P= (2, -3, 4) and perpendicular to the line defined by the points a = (1, 5, 7) and b =(-2, 2, 3). ***(2 marks)***

   b) For what value of $\lambda$ are the vectors $\vec{i}+2\vec{j}-3\vec{k}$, $3\vec{i}+\lambda\vec{j}+\vec{k}$ and $\vec{i}+2\vec{j}+3\vec{k}$ coplanar? ***(2 marks)***

7. Consider the following figure:




   ABCD is a square, CE=AE, $\overline{BD}$ = x. Find sin $\alpha$, cos $\alpha$ and tan $\alpha$. ***(3 marks)***

8. a) Show that ${}^{n-1}C_{p-1}+{}^{n-1}C_p={}^nC_p$, $0\le P\le n$. ***(3 marks)***

   b) Solve the equation ${}^{n-1}C_{n-5}=3^{n-3}C_{n-7}$ in the set of positive integers. ***(2 marks)***

9. Evaluate the following limits:

   a) $\lim\limits_{x\to\pm\infty}\sqrt{x^2+x+2}-\sqrt{x^2-x+3}$ ***(3 marks)***

b) $\lim_{x\to 0} \frac{x^2 - x\sin x}{x^2 - \sin^2 x}$ **(2 marks)**

10. Let $Z_1 = -1 + i$ and $Z_2 = -\sqrt{2} - \sqrt{6}\,i$

a) Find the trigonometric forms of $Z_1$ and $Z_2$. **(3 marks)**

b) Write the product $Z_1.Z_2$ in Cartesian and trigonometric form. **(3 marks)**

11. Four ships A, B, C and D are at sea in the following relative positions:

B is on the straight line segment AC, B is due North of D and D is due West of C. The distance between B and D is 2 km. $\prec BDA = 40°$, $\prec BCD = 25°$.

What is the distance between A and D? **(2 marks)**

12. A conic is given by the polar equation $r = \frac{10}{3 - 2\cos\theta}$

Find the eccentricity, identify the conic, locate its directrix, and sketch the conic. **(4 marks)**

13. If $y = e^x \cot x$, find the value of $\frac{dy}{dx} - y + e^x \operatorname{cosec}^2 x$. **(2 marks)**

14. Calculate $\int_0^{\pi/2} \cos^3 x\, dx$. **(2 marks)**

15. Consider the numerical function $f$ defined by $f(x) = \frac{1}{\ln x}$

a) Calculate the first derivative of $f$ on $(o, +\infty)$. **(3 marks)**

b) If $n \geq 2$, calculate $In = \int_e^{e^n} \frac{1}{8x(lnx)^2}\, dx$ **(2 marks)**

c) Calculate $\lim_{n\to +\infty} In$ **(1 mark)**

**SECTION B: ATTEMPT THREE QUESTIONS ONLY. (45 marks)**

16. a) Consider a sample space S on which the probability P is defined. Consider also two events A and B such that: $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{4}$, $P(A) = \frac{2}{3}$

Find: P(B), $P(\overline{A})$, $P(\overline{B})$ and $P(\overline{A \cap B})$ **(5 marks)**

b) A fair coin is tossed 3 times. Find the probability for obtaining two heads. **(4 marks)**

c) A factor has three machines A, B, C producing large numbers of a certain items. Of the total daily production of the items, 50% are produced on A, 30% on B and 20% on C. Records show that 2% of items produced on A are defective, 3% of items produced on B are defective and 4% of items produced on C are defective. The occurrence of a defective item is independent of all other items. One item is chosen at random from a day's total output.

(i) Show that the probability of its being defective is 0.027 **(4 marks)**

(ii) Given that it is defective, find the probability that it was produced on machine A. **(2 marks)**

17. Let $f(x) = \dfrac{x^3}{x^2-1}$

a) Determine the domain of definition of $f(x)$. **(1 mark)**

b) Calculate the limits around the domain of definition and deduce the equations of the asymptotes to the curve of $f(x)$. **(3 marks)**

c) Find the coordinate of intersection points of the asymptotes to the curve of $f(x)$. **(1 mark)**

d) Study the variation sense of f and draw the variation table. **(4 marks)**

e) Determine the equations of tangent and normal to the curve at the point of inflection (if it exists). **(2 marks)**

f) Determine the coordinates of the points of intersection of the curve with axis coordinates. **(2 marks)**

g) Sketch the graph of f(x). **(2 marks)**

18. a) An aeroplane flying horizontally 1 km above the ground is observed at the elevation of $60^0$. If after 10 seconds the elevation is observed to be $30^0$, find the uniform speed per hour of the aeroplane. **(9 marks)**

b) Two sides of a triangle have lengths of 50 cm and 80 cm respectively. The measure of their included angle is $60^0$.
Find the length of the side opposite to $60^0$ approximately. **(6 marks)**

19. a) Find the equation of parabola whose focus is at (-1, -2) and directrix $x - 2y + 3 = 0$. **(4 marks)**

b) Find the equation of the set of the all points whose distances from (0, 4) are $\frac{2}{3}$ of their distances from the line $y = 9$. **(5 marks)**

c) In the hyperbola $x^2 - 4y^2 = 4$, find the axes, the co-ordinates of the foci, the eccentricity and the latus rectum. **(6 marks)**

20. A random variable x has probability density function

$$F(x) = \begin{cases} Ax(6-x)^2; 0 \le x \le 6 \\ 0 \text{ elsewhere} \end{cases}$$

a) Find the value of the constant A. **(4 marks)**

b) Calculate :

(i) The mean **(5 marks)**

(ii) The variance **(4 marks)**

(iii) The standard deviation **(2 marks)**